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Institute of Mathematics

Aspects of the global theory of minimal surfaces and harmonic maps

Institute Colloquium

16.02.2022

Online Meeting

14:00 Lynn Helle, Leibniz University, Hannover)

Minimal and constant mean curvature surfaces in the 3-sphere

Minimal or, more generally, constant mean curvature surfaces can be viewed as optimal realizations of a compact (Riemann) surface into the ambient space — in our case the round 3-sphere. Though minimal surfaces of any genera was shown to exist by Lawson in 1970, computing geometric properties of these surfaces has been very challenging. In particular, the area of these minimal surfaces, except for the trivial cases of spheres and tori, are not known. In my talk I want to present an alternate way of constructing Lawson surfaces of high genus using a generalization of the famous Weierstrass respresentation of minimal surfaces in the Euclidean 3-space (joint work with Sebastian Heller and Martin Traizet). This approach also allows us to find the first complete families of constantmean curvature surfaces of large genus deforming the Lawson surface. Moreover, we obtain iterative algorithms to compute the power series expansion of the area of these surfaces. Surprisingly, the coefficients have a closed form solution in terms of multiple-polylogarithms and at least the lower order terms can be identified with particular values of the Riemann zeta function.

15:00 Markus Röser, (University of Hamburg)

Harmonic maps and the Deligne--Hitchin twistor space

Harmonic maps are critical points of the Dirichlet energy functional between two manifolds. Harmonic maps from a Riemann surface into a symmetric space may be described by a gauge-theoretic formalism which provides a link between such maps and families of flat complex connections. When the target is the three-dimensional hyperbolic space, (twisted) harmonic maps are described by pairs consisting of a connection and a so-called Higgs field solving Hitchin's self-duality equations. The family of flat connections induced by a harmonic map can then be interpreted as a holomorphic section of the so-called Deligne--Hitchin twistor fibration, satisfying a certain reality condition. It turns out that in a similar way, we may think of (twisted) harmonic maps into the three-sphere as holomorphic sections of the Deligne--Hitchin fibration. These observations provided the motivation to explore the geometry of the space of holomorphic sections of the Deligne--Hitchin fibration. In this talk, which is based on joint work with Florian Beck, Indranil Biswas and Sebastian Heller, we will discuss a natural functional on the space of holomorphic sections generalising the Dirichlet energy functional for harmonic maps.